The deflection of the jet at the trailing edge is

$$y_{j_1}'(c) = \alpha_1 + \tau_1 \quad (4)$$

and at infinity

$$y_{j_1}'(\infty) = 0 \quad (5)$$

The solution of Eqs. (1–5), Ref. (1), gives an expression for the derivative of the jet shape of the form

$$y_{j_1}'(x) = \alpha_1 F(x,C_{j_1}) + \tau_1 G(x,C_{j_1}) \quad (6)$$

where C_{j_1} is the jet momentum coefficient defined as the jet momentum per unit span divided by the product of the wing chord and freestream dynamic pressure. The lift coefficient may be written

$$C_{L_1} = \tau_1 \partial C_{L_1}/\partial \tau_1 + \alpha_1 \partial C_{L_1}/\partial \alpha_1 \quad (7)$$

where for $0 < C_{j_1} < 10$ the following formulas for the lift derivatives are applicable

$$\partial C_{L_1}/\partial \tau_1 = 2(\pi C_{j_1})^{1/2}(1 + 0.151 C_{j_1}^{1/2} + 0.139 C_{j_1})^{1/2} \quad (8)$$

$$\partial C_{L_1}/\partial \alpha_1 = 2\pi(1 + 0.151 C_{j_1}^{1/2} + 0.219 C_{j_1}) \quad (9)$$

Formulation of Similarity Rule

Following Ref. 2, consider the potential function $\Phi(\xi,\eta)$ of a compressible flow in (ξ,η) coordinates having freestream Mach number M and freestream velocity U_2. Choose Φ to be related to ϕ by

$$\phi(x,y) = A(U_1/U_2)\Phi(\xi,\eta) \quad (10)$$

$$\xi = x,\ \eta = y/(1 - M^2)^{1/2} \quad (11)$$

where A is a constant. Introducing Eqs. (10) and (11) into Eq. (1) yields

$$\partial^2\Phi/\partial\xi^2 + [1/(1 - M^2)]\partial^2\Phi/\partial\eta^2 = 0 \quad (12)$$

Hence Φ is a solution corresponding to Mach number M. The boundary conditions (2) and (3) yield

$$(\partial\Phi/\partial\eta)_{\eta=0} = U_2\alpha_2 \qquad 0 < \xi < c \quad (13)$$

$$(\partial\Phi/\partial\eta)_{\eta=0} = U_2[\alpha_2 F(\xi,BC_{j_2}) + \tau_2 G(\xi,BC_{j_2})] \qquad c < \xi < \infty \quad (14)$$

provided

$$\alpha_1(1 - M^2)^{1/2}/A = \alpha_2,\quad \tau_1(1 - M^2)^{1/2}/A = \tau_2,\quad C_{j_1} = BC_{j_2} \quad (15)$$

It is shown below that B can be chosen so as to make the bracketed term of Eq. (14) equal to the derivative of the jet shape in the (ξ,η) plane. Thus, Eq. (14) becomes

$$(\partial\Phi/\partial\eta)_{\eta=0} = U_2 y_{j_2}'(\xi) \qquad c < \xi < \infty \quad (16)$$

Equations (4) and (5) are similarly transformed.

The pressure coefficient of the incompressible solution may be related to the pressure coefficient of the compressible solution by

$$C_{p_1} = -(2/U_1)(\partial\phi/\partial x)_{y=0} = -(2A/U_2)(\partial\Phi/\partial\xi)_{\eta=0} = AC_{p_2} \quad (17)$$

A normal momentum balance about the jet yields the following relationship between the pressure discontinuity across the jet and its radius of curvature

$$\Delta C_p = -cC_j/R \quad (18)$$

where R is the radius of curvature. Thus the radius of curvature of the jet shapes in the two flow systems are related by

$$R_2 = -cC_{j_2}/\Delta C_{p_2} = -cC_{j_1}A/B\Delta C_{p_1} = AR_1/B \quad (19)$$

It is easily demonstrated, within the approximations of thin-airfoil theory, that

$$1/R_1 = -d^2y_{j_1}/dx^2,\ 1/R_2 = -d^2y_{j_2}/d\xi^2 \quad (20)$$

Combine the two preceding expressions withEqs. (6, 15, and 19) to obtain

$$y_{j_2}'(\xi) = [B/(1 - M^2)^{1/2}][\alpha_2 F(\xi,BC_{j_2}) + \tau_2 G(\xi,BC_{j_2})] \quad (21)$$

Imposing the condition $B = (1 - M^2)^{1/2}$ verifies Eq. (16). Owing to the homogeneity of the differential equations the constant A remains arbitrary. The similarity law expressed by Eqs. (15) and (17) may be written in the general form

$$C_p/A = f[\alpha/A(1 - M^2)^{1/2},\ \tau/A(1 - M^2)^{1/2},\ C_j(1 - M^2)^{1/2}] \quad (22)$$

Choosing $A = 1$ the incompressible solution can be generalized to a subsonic flow by replacing α_1, τ_1, and C_{j_1} appearing in the incompressible expressions by $\alpha/(1 - M^2)^{1/2}$, $\tau/(1 - M^2)^{1/2}$, and $C_j(1 - M^2)^{1/2}$, respectively.

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Downwash Correction for a Two-Dimensional Finite Wing

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A FINITE two-dimensional wing in a wind tunnel behaves as a three-dimensional wing because of the downwash induced by tip and wall vortices. For design work the true two-dimensional flow over the wing section is desired and much effort is usually expended in test programs to correct for the downwash. In the discussion to follow it is demonstrated that the true two-dimensional flow over the two-dimensional finite wing can be found without the need to correct, experimentally, for the downwash provided the pressure distribution over the section is known from test data.

As an illustration of the method consider the cambered ellipse wing section shown in Fig. 1. Its wing span was 16.5 in. and it was tested in the West Virginia University subsonic tunnel at a geometric angle of attack $\alpha_g = 10°$ and a freestream velocity $V_\infty = 150$ fps. The cambered ellipse section had a thickness-chord ratio of 20%, a 5% camber, and a chord $c = 9$ in.; 37 pressure taps were arranged around its periphery at the wing midspan. Vectored blowing was introduced at the blowing slot and the pressure distribution shown by the open encircled points in Fig. 1 was obtained.[1] This test was part of a continuing program to study the feasibility of obtaining high-lift devices by vectored blowing around bluff-ended bodies.[2,3]

From the test data the normal sectional coefficient c_n and the chordwise sectional coefficient c_h are obtained from the

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expressions:

$$c_n = \int_0^1 (c_{pl} - c_{pu}) d\left(\frac{\xi}{c}\right) \tag{1}$$

$$c_h = \int_0^1 \left[c_{pu}\left(\frac{dY}{dX}\right)_u - c_{pl}\left(\frac{dY}{dX}\right)_l \right] d\left(\frac{\xi}{c}\right) \tag{2}$$

where $c_p = (p - p_\infty)/\frac{1}{2}\rho V_\infty^2$, the pressure coefficient; the subscript $(\ )_u$ refers to the upper surface $Y > 0$, the subscript $(\ )_l$ refers to the lower surface $Y < 0$, and ξ is the distance along the X axis from the leading edge.

By resolving c_n and c_h normal to the resultant velocity V, the test lift coefficient is expressed by

$$(c_l)_{\text{TEST}} = c_n \cos\alpha_e - c_h \sin\alpha_e \tag{3}$$

which contains the two unknowns α_e and c_l.

Another equation between α_e and c_l is obtained from Theodorsen's method[4] which provides a one-to-one relation between each point (X,Y) on the airfoil section to each point (a,Φ) on the circle (Fig. 1). Theodorsen's method also provides the equation for the velocity at each point on the airfoil.[4] For the present discussion the complete equation is not required, but it is sufficient to note that the velocity at each point on the airfoil $V_a(X,Y)$ is related to its corresponding point (a,Φ) on the circle by

$$V_a(X,Y) = F \cdot \left| \frac{c_l c}{8\pi a} + \sin(\Phi - \alpha_e) \right| \tag{4}\dagger$$

The test data provides the point on the airfoil where $c_p = +1$. This is the forward stagnation point and the related angle Φ_{stag} on the circle is known from the Theodorsen transformation.[4] At the stagnation point $V_a = 0$, therefore, Eq. (4) provides for the theoretical lift coefficient

$$(c_l)_{\text{THEORETICAL}} = -(8\pi a/c)\sin(\Phi_{\text{stag}} - \alpha_e) \tag{5}$$

Equating relations (3) and (5), the effective angle of attack is calculated from

$$\tan\alpha_e = [c_n + (8\pi a/c)\sin\Phi_{\text{stag}}]/[c_h + (8\pi a/c)\cos\Phi_{\text{stag}}] \tag{6}$$

The lift coefficient follows from either Eq. (3) or (5), the induced angle of attack from $\alpha_i = \alpha_g - \alpha_e$, and the resultant velocity V from $V = V_\infty \sec\alpha_i$.

The present method was applied to the test points in Fig. 1 and gave $\alpha_e = 6.98°$, $c_l = 2.362$. The theoretical pressure

† A similar relation results from the Joukowski transformation; however the function F is different for the two transformations. For the cambered ellipse in Fig. 1 the Theodorsen transformation is required.

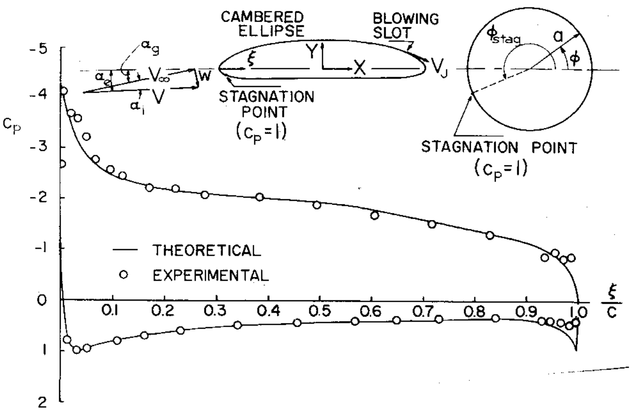

Fig. 1 Comparison of experimental data with theoretical analysis.

distribution on the cambered ellipse (Fig. 1) for these values of α_e and c_l was obtained by using Theodorsen's method to calculate the velocity and Bernoulli's equation to calculate the pressure. The theoretical curve follows the test points rather closely except near the leading edge on the upper surface which seems to indicate (because of the high geometric angle of attack) a laminar separation bubble, and near the trailing edge. In the latter region it is known that vectored blowing affects the theoretical pressure distribution.[1,2] However, since the test c_l and the theoretical c_l are equal, the area under the theoretical curve is equal to the area under the experimental points in Fig. 1.

The method proposed herein applies to airfoil sections of any shape with or without blowing (or suction) provided the spanwise flow at the section where the experimental pressure distribution is obtained is negligible. It is also assumed that the contribution of the blowing (or suction) momentum to the lift is negligible.

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